UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2005

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

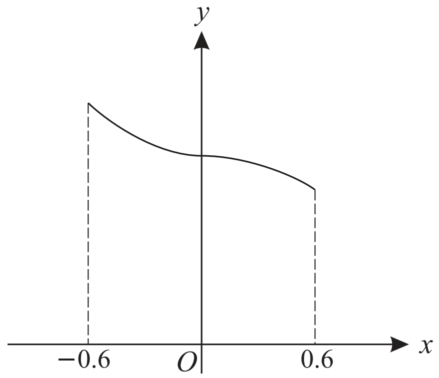
This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Expand $(1 + 4x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

2



The diagram shows a sketch of the curve $y = \dfrac{1}{1 + x^3}$ for values of x from -0.6 to 0.6.

(i) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_{-0.6}^{0.6} \frac{1}{1 + x^3}\, dx,$$

giving your answer correct to 2 decimal places. [3]

(ii) Explain, with reference to the diagram, why the trapezium rule may be expected to give a good approximation to the true value of the integral in this case. [1]

3 **(i)** Solve the equation $z^2 - 2iz - 5 = 0$, giving your answers in the form $x + iy$ where x and y are real. [3]

(ii) Find the modulus and argument of each root. [3]

(iii) Sketch an Argand diagram showing the points representing the roots. [1]

4 **(i)** Use the substitution $x = \tan\theta$ to show that

$$\int \frac{1 - x^2}{(1 + x^2)^2}\, dx = \int \cos 2\theta \, d\theta. \qquad [4]$$

(ii) Hence find the value of

$$\int_0^1 \frac{1 - x^2}{(1 + x^2)^2}\, dx. \qquad [3]$$

5 The polynomial $x^4 + 5x + a$ is denoted by $\mathrm{p}(x)$. It is given that $x^2 - x + 3$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a and factorise $\mathrm{p}(x)$ completely. [6]

(ii) Hence state the number of real roots of the equation $\mathrm{p}(x) = 0$, justifying your answer. [2]

6 **(i)** Prove the identity

$$\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3.$$ [4]

(ii) Hence solve the equation

$$\cos 4\theta + 4\cos 2\theta = 2,$$

for $0° \leqslant \theta \leqslant 360°$. [4]

7 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\operatorname{cosec} x = \tfrac{1}{2}x + 1,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify, by calculation, that this root lies between 0.5 and 1. [2]

(iii) Show that this root also satisfies the equation

$$x = \sin^{-1}\left(\frac{2}{x+2}\right).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \sin^{-1}\left(\frac{2}{x_n+2}\right),$$

with initial value $x_1 = 0.75$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 **(i)** Using partial fractions, find

$$\int \frac{1}{y(4-y)}\,\mathrm{d}y.$$ [4]

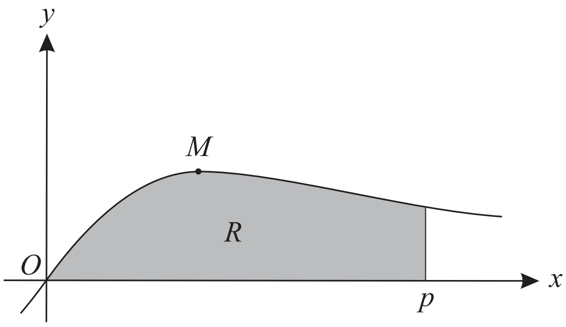
(ii) Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y(4-y),$$

obtaining an expression for y in terms of x. [4]

(iii) State what happens to the value of y if x becomes very large and positive. [1]

9



The diagram shows part of the curve $y = \dfrac{x}{x^2 + 1}$ and its maximum point M. The shaded region R is bounded by the curve and by the lines $y = 0$ and $x = p$.

(i) Calculate the x-coordinate of M. [4]

(ii) Find the area of R in terms of p. [3]

(iii) Hence calculate the value of p for which the area of R is 1, giving your answer correct to 3 significant figures. [2]

10 With respect to the origin O, the points A and B have position vectors given by

$$\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation $\mathbf{r} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + s(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$.

(i) Prove that the line l does not intersect the line through A and B. [5]

(ii) Find the equation of the plane containing l and the point A, giving your answer in the form $ax + by + cz = d$. [6]